

## **Restricted Stock Studies: Estimating Discount for Lack of Marketability**

Recent publication of the Discount for Lack of Marketability(DLOM) Job aid for IRS Valuation Professionals has brought fresh scrutiny to the various methods used by valuation practitioners for determining discount for lack of marketability. One area of emphasis in the job aid is the use of various regression models for developing DLOM estimates. This paper performs an evaluation of three widely used regression models and presents a best fit model identifying variables that may be useful in developing DLOM estimates.

Revenue ruling 77-287 is the primary authority relied upon by valuation practitioners for using the restricted stock studies method for estimating a discount for lack of marketability. This ruling, is influenced by the United States Securities & Exchange Commission (SEC) guidelines provided in Investment Company Act Release No. 5847, dated October 21, 1969, which stated that " there can be no automatic formula by which an investment company can value the restricted securities in its portfolios. Rather, the SEC has determined that it is the responsibility of the board of directors of the particular investment company to determine the "fair value" of each issue of restricted securities in good faith." IRS revenue ruling 77-287 refers to "restricted securities," also known variously as "unregistered securities," "investment letter stock," "control stock," or "private placement stock." Frequently these terms are used interchangeably. These designations all indicate that these particular securities cannot lawfully be distributed to the general public until a registration statement relating to the corporation underlying the securities has been filed, and has also become effective under the rules promulgated and enforced by the SEC. These trading restrictions are primarily designed to protect the non-insider trading public.

The very first organized study of the differences observed between prices of freely traded stocks and their counterpart restricted stocks was undertaken by the SEC pursuant to Congressional direction. It was a survey study, covering the restricted stocks issued during January 1, 1966, through June 30, 1969. The original report, comprising of eight volumes, was published in March 1971. Fifth volume of this eight volume series

provides an analysis of restricted securities and deals with such items as the characteristics of the restricted securities purchasers and issuers, the size of transactions (dollars and shares), the marketability discounts on different trading markets, and the resale provisions applicable to the surveyed company issues. The study results indicated that the following factors were influential in determining the size of the marketability discount.

(a) Earnings. Earnings and sales consistently have a significant influence on the size of restricted securities discounts according to the study. Earnings played the major part in establishing the ultimate discounts at which these stocks were sold from the current market price.

(b) Sales. The dollar amount of sales of issuers' securities also has a major influence on the amount of discount at which restricted securities sell from the current market price.

(c) Trading Market. The market in which publicly held securities are traded also reflects variances in the amount of discount that is applied to restricted securities purchases.

(d) Resale Agreement Provisions. Resale agreement provisions often affect the size of the discount.

In conclusion, the study suggested that "Earnings, net assets, and net sales must be given primary consideration in arriving at an appropriate discount for restricted securities from the freely traded shares." These factors have been used as the building blocks of empirical analysis for determining the discount for lack of marketability using the restricted stock studies method.

Historically, two approaches have developed in this analytical method. A simpler approach has been the benchmarking approach, using compilation and descriptive statistics (Mean, Median) of the observed discounts and using these numbers as indicative of the lack of marketability discount applied to the subject of valuation. Generally these studies do not provide many data points, and present statistical challenges

because of a wide dispersion in the samples, standard deviations of the reported discounts are very large, reducing their statistical robustness. The continuing emphasis by tax courts on rigor of analysis has been pushing the valuation profession towards more quantitative testable methods. Recently, there has been an increasing tendency toward using the various regression models developed over time to determine an applicable discount for lack of marketability. This paper presents tests of three regression models widely quoted in valuation literature, and provides a new model identifying variables that are found to be significant.

It is important to start with a note of caution. The restricted stocks ‘studies’ were designed to be descriptive, rather than predictive models. Therefore, the results of prior regression studies **do not provide a plug and play method** for determining discounts for lack of marketability, as suggested by some practitioners. While the results are indicative of the factors that are influential in determining the applicable discount for lack of marketability, it is important that the sample used for determining the applicable discount for a particular subject be as closely comparable to the subject interest. Most commonly cited regression models for determining the discount for lack of marketability are

Study	Period	Sample size	Observed mean discount
Silber	1981-88	69	33.75%
Hertzel Smith	1980-87	106	20.14%
Bajaj	1990-95	88	7%

These studies identify five categories of variables significant in explaining the observed discounts

- (A) Firm Size : measured as Market Value, Revenues, Working Capital, Common Equity, Book Value
- (B) Earning Volatility: measured as Annual Variance, Standard Deviation of daily returns
- (C) Block Size: measured as Percentage placed
- (D) Holding/Liquidation Period: measured as Reg 144 requirements, and

(E) Financial Distress: measured as Z-score, Adverse News

These Restricted stock studies describe the relationship between the observed price differential between restricted and unrestricted stock prices. These studies assume Liquidity and do not consider any price pressure impact from selling a large block of stock in the market place. These models were re-estimated using a dataset created from FMV database, CRSP, and Compustat databases.

### **Data Sample description**

In order to create a data set comparable to the ones used for the different studies, I started with the FMV Opinions database. This database covers restricted stock transactions occurring during 1980-2005, and provides detailed descriptors for the reported transactions. In addition, the data were cross tabulated with CRSP and Compustat data sets, to identify transactions for which the following information was available to create datasets compatible with Silber, H&S, and Bajaj studies

(A) Market Value, Book Value of Equity,

(B) Z Score factors ( EBIT, Total Retained Earnings, Total Assets, and Total Liabilities)

(C) Revenues, Net income, and

(D) Daily Returns for one year prior to the issue of restricted stock

This procedure resulted in data set of 151 transactions for which all variables were available for at least one of the models. The number of observations usable for each model was smaller because of individual missing values. This enabled me to test the models using the same base data for all models.

**Regression Results:** Results for the three historically specified models are provided in the following Tables 1-3. It is interesting to note that Block size, while proposed in each model, is not found to be significant in any of the determinations using this dataset. This is not surprising, as the regulatory regime has changed from an initial restriction period of two years to one year during the period covered by the sample,

potentially confounding the two effects. As expected, liquidation period, which is the result of the block size and regulatory restrictions, is significant in explaining the observed discounts. Firm Size, financial distress, and risk, measured by standard deviation of returns are also found to be significant in explaining the size of the observed discount. The adjusted regression r squares range from 2% (Bajaj) to 17% (Silber). An alternative model was also estimated. Formerly identified model factors were tested using the same base data to find the best fit model with significant explanatory power. I tested different specifications for size, volatility, liquidation period, financial distress, and registration status. The model presented in table 4, uses the size (revenues), standard deviation of returns, liquidation period, and financial distress(Z-score) for the company as explanatory factors. The results indicate that this group of variables may be the best predictive combination for the sample. As expected, a larger firm (revenues), with better financial health (z-score), stable returns( low standard deviation), and shorter liquidation period, faces a smaller discount. The model provides a better fit, with an adjusted r square of 35%.

It is necessary here to emphasize that while these regression models provide a test of influential factors, they do not provide a 'Plug and Play' matrix for determining DLOM. At best a regression model estimate can provide a confidence interval for the applicable discount with a pre-determined level of certainty, e.g. a 95% likelihood that the real discount lies between the two end values. Larger, more uniform samples are likely to yield tighter, more useful estimates. It is desirable to start with an open source dataset which allows the appraiser to select a justifiable comparison sample, estimate coefficients, and use the model results to help in estimating applicable range of discounts. When using regression models it is important to realize that r square, while useful, is not the sole measure of comparison among models. In addition, the regression coefficients for individual factors should be significant (t test), and the F-statistic for the regression should be significant. In general when comparing models, models with significant variable t statistics, higher regression F and adjusted r square are preferred. It is also necessary to emphasize that this estimate of the discount for lack of marketability incorporates only the delayed liquidation due to regulatory constraints, but does not measure the likely price pressure from the blockage perspective. In order to make a complete

determination of the discount it may be useful to take the likely blockage effect into account. Amihud (2002) used the ratio of absolute stock returns to dollar volume as a liquidity measure, following Kyle's concept of liquidity. This ratio, frequently referred to as Amihud's ratio, can be used to estimate the incremental impact of liquidating the block at the expiration of the regulatory restrictions on the restricted block of stock being evaluated.

Table 1. Silber Model Estimation

Dependent Variable: log ( 1-Discount)

Number of Observations Read 151

**Number of Observations Used 141**

Number of Observations with Missing Values 10

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	1.03851	0.34617	10.68	<.0001
Error	137	4.44116	0.03242		
Corrected Total	140	5.47			
Root MSE	0.18005	R-Square	0.1895		
Dependent Mean	4.35785	Adj R-Sq	0.1718		
Coeff Var	4.13158				

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr >  t
Intercept	1	4.19992	0.04849	86.61	<.0001
<b>ln(revenue)</b>	<b>1</b>	<b>0.03924</b>	<b>0.00720</b>	<b>5.45</b>	<b>&lt;.0001</b>
ln (Block size)	1	-0.02301	0.01522	-1.51	0.1328
Positive Earning	1	-0.01412	0.03501	-0.40	0.6874

Size measured as log of revenues is the only factor found to be statistically significant.

Table 2. Hertzels Smith Estimation

Dependent Variable: Discount Observed

Number of Observations Read	151
Number of Observations Used	148
Number of Observations with Missing Values	3

Analysis of Variance

Source	Sum of		F Value	Pr > F
	DF	Squares		
Model	5	0.29683	3.23	0.0085
Error	142	2.60621		
Corrected Total	147	2.90304		
Root MSE		0.13548	R-Square	0.1022
Dependent Mean		0.19719	Adj R-Sq	0.0706
Coeff Var		68.70235		

Parameter Estimates

Variable	DF	Parameter		t Value	Pr >  t
		Estimate	Standard Error		
Intercept	1	0.21704	0.02412	9.00	<.0001
Block size	1	0.15445	0.14582	1.06	0.2913
<b>Financial Distress</b>	<b>1</b>	<b>-0.06281</b>	<b>0.02416</b>	<b>-2.60</b>	<b>0.0103</b>
<b>Market value</b>	<b>1</b>	<b>-4.74919E-8</b>	<b>1.597023E-8</b>	<b>-2.97</b>	<b>0.0035</b>
Book TO Market	1	4.27690	29.89900	0.14	0.8865
Registration Status	1	0.00316	0.02379	0.13	0.8946

Size measured as market value and Financial distress are significant variables.



Table 3. Bajaj Model Estimation

Dependent Variable: Observed Discount

Number of Observations Read	151
Number of Observations Used	132
Number of Observations with Missing Values	19

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	0.14259	0.03565	1.73	0.1484
Error	127	2.62406	0.02066		
Corrected Total	131	2.76665			

Root MSE 0.14374 R-Square 0.0515

Dependent Mean 0.20251 Adj R-Sq 0.0217

Coeff Var 70.97970

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr >  t
Intercept	1	0.00911	0.07548	0.12	0.9042
Block size	1	0.22771	0.16785	1.36	0.1773
Z-score	1	0.00016969	0.00029912	0.57	0.5715
Registration Status	1	0.00795	0.02661	0.30	0.7656
<b>Std Deviation of ret</b>	<b>1</b>	<b>0.18101</b>	<b>0.07517</b>	<b>2.41</b>	<b>0.0175</b>

**Riskiness measured as Standard deviation of returns is the only significant variable.**

Table 4. Best Fit Estimated Model FMV Data

Dependent Variable: ln (1-Discount)

Number of Observations Read	151
Number of Observations Used	102
Number of Observations with Missing Values	49

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	1.40891	0.35223	14.58	<.0001
Error	97	2.34381	0.02416		
Corrected Total	101	3.75273			

Root MSE	0.15544	R-Square	0.3754
Dependent Mean	4.33993	Adj R-Sq	0.3497
Coeff Var	3.58173		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr >  t
Intercept	1	4.44154	0.12161	36.52	<.0001
<b>ln(Revenue)</b>	<b>1</b>	<b>0.04184</b>	<b>0.00727</b>	<b>5.75</b>	<b>&lt;.0001</b>
<b>ln(Std. Dev. Ret)</b>	<b>1</b>	<b>-0.27612</b>	<b>0.09260</b>	<b>-2.98</b>	<b>0.0036</b>
<b>ln (Liquidation Period)</b>	<b>1</b>	<b>-0.09803</b>	<b>0.03700</b>	<b>-2.65</b>	<b>0.0094</b>
<b>ln (z-score )</b>	<b>1</b>	<b>0.02590</b>	<b>0.01221</b>	<b>2.12</b>	<b>0.0365</b>

**All four variables, Size, Riskiness, Liquidation period, and financial condition, are significant.**

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